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# The added mass of a flexible plate oscillating in a fluid

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**Abstract**

This work presents a review and a numerical study of the fundamental properties of the added mass of a plane flexible plate oscillating in a fluid. A low aspect ratio plate is immersed in a stationary fluid. The plate is clamped at one edge and free at the other edges. The plate vibrates in a single natural mode. Thin airfoil theory is applied to calculate the pressure jump across the plate. The magnitude of the added mass is calculated for a spanwise half-sine fundamental mode and the first 10 natural chordwise modes for plates with low aspect ratio. It is found that an increase of the order of the mode of vibration decreases the added mass. A decrease of the aspect ratio ($A$) leads to a decrease of the added mass, and the attenuation of the dependence on the order of the chordwise natural mode**.** The numerical results show that the noticeable dependence of the added mass on the order of the chordwise natural mode diminishes as $A \rightarrow 0.01$. For $A = 1.0$, the results obtained by three-dimensional theory resemble the results obtained by two-dimensional version of the basic solutions ($A = \infty$). For the verification of the present method, calculations were carried out also for the simply supported plate. It is shown that the obtained data are in good agreement with known results.


## 1. Introduction

The concept of added mass has a wide application in the dynamic analysis of a flexible plate that is subjected to fluid flow, in particular, in flutter analysis (Datta and Gottenberg, 1975). The problem of the flexible plate flutter has a solution when the fluid loading caused by motion of the plate is known. The added mass is a characteristic of the fluid loading (Pretlove, 1965; Minami, 1998). The understanding of the properties of the added mass will allow the fluid loading to be obtained in the most suitable form. Some determinations of the fluid loading and the added mass for a supported plate are known from the slender wing theory (Jones, 1946), the travelling wave solution (Miles, 1956; Dugundji et al., 1963), two-dimensional linear aerodynamic theory (Kornecki et al., 1976), or three-dimensional linear aerodynamic theory (Lucey and Carpenter, 1993). The existing knowledge regarding the added mass is limited to the specific cases mentioned above, and is not applicable to a wide range of low aspect ratio cantilever plates.

The fluid loading on a cantilever plate in an axial flow may be presented as the sum of a noncirculatory part, and a circulatory part (Kornecki et al., 1976). The noncirculatory part of the fluid loading has the effects of modifying the added mass, the fluid dynamic damping and the added stiffness on the plate. The added mass is present in the noncirculatory part only, but the fluid damping appears in both the noncirculatory and circulatory parts (Huang, 1995).

Minami (1998) has considered a membrane that has its ends fixed in an incompressible fluid within the framework of the thin airfoil theory. He has shown that the added mass does not depend on the frequency and the amplitude of the oscillations; the added mass uniquely depends on $\rho l$, and for the first mode of oscillation it is expressed as $M = 0.68\rho l$. Here $\rho$ and $l$ are the fluid density and the membrane length, respectively.

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**Nomenclature**

| | |
|---|---|
| $A$ | aspect ratio, $H/l$ |
| $a(t)$ | function of time |
| $E$ | kinetic energy of the fluid |
| $F_I, F_D, F_S$ | aerodynamic inertia, damping and stiffness |
| $H$ | plate width |
| $h$ | surface displacement in normal direction |
| $h_1$ | chordwise deflection's coordinate |
| $h_2$ | spanwise deflection's coordinate |
| $k$ | wave number, $2\pi/\lambda$ |
| $l$ | plate length |
| $M$ | added mass of a surface plate per unit area |
| $n$ | chordwise mode number |
| $\Delta p$ | fluid loading |
| $s$ | curvilinear coordinate along the deforming plate |
| $t$ | time |
| $U$ | freestream fluid velocity |
| $V_N$ | fluid particle velocity in the direction of the local normal $N$ |
| $x, y, z$ | Cartesian coordinates in the deformed state of plate |
| $\bar{x}, \bar{y}$ | dimensionless coordinates, $x/l$, $y/H$ |
| $\beta_n$ | $n$th eigenvalue |
| $\lambda$ | wavelength |
| $\mu$ | nondimensional value of the added mass, $M\pi/\rho l$ |
| $\rho$ | fluid density |
| $\sigma$ | surface of fluid region |
| $\Phi_n(\bar{x})$ | $n$th natural mode of the plate oscillation in $x-z$ plane |
| 2-D | two-dimensional |
| 3-D | three-dimensional |

This review suggests that attention up to now has been restricted to few works and has mainly focused on the added mass of a simply supported plate oscillating in still fluid.

## 2. Definition of the added mass

### 2.1. Model of oscillating plate

In a static state, a plate having length $l$ and width $H$, is fixed at the leading edge and free at the other edges. The plate is a very thin flexible surface having a low aspect ratio. The $(x, y, z)$ coordinate system, having the origin at the leading edge of the plate, is shown in Fig. 1. The $(x, y)$-axes are oriented in the direction of the length and the width, respectively, of the plate.

In the dynamic state, the plate immersed in the fluid is assumed to be undergoing a natural oscillation continuously in the half-sine standing wave fundamental mode in the $y-z$ plane and in the one standing wave natural mode in the $x-z$ plane. The expression for the displacement $h$ at any point $x, y$ and time $t$ is (Ellen, 1973)

$$h(\bar{x}, \bar{y}, t) = a(t)\Phi_n(\bar{x})\sin \pi\bar{y}, \quad 0 \leqslant \bar{x} \leqslant 1, \; 0 \leqslant \bar{y} \leqslant 1, \; n = 1, 2, 3, \ldots, \tag{1}$$

where $a(t)$ is a function of time, $\Phi_n(\bar{x})$ is the $n$th natural mode of the plate oscillation, $\bar{y} = y/H$, $\bar{x} = x/l$ are spanwise and streamwise nondimensional coordinates along the plate, respectively.

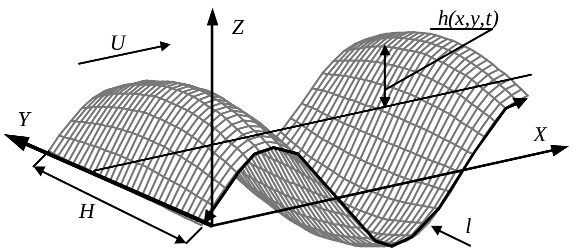


Fig. 1. Model of the cantilever plate.

### 2.2. Determination of the added mass

In the case of an incompressible inviscid stationary fluid the rate of change in time of kinetic energy of any portion of the fluid is equal to the work done by the pressures on its surface (Lamb, 1932):

$$\frac{\mathrm{d}E}{\mathrm{d}t} = -\int\int_{\sigma} V_N \Delta p \,\mathrm{d}\sigma, \tag{2}$$

where $E$ is the kinetic energy, $\sigma$ is the surface bounding the fluid region and $V_N$ denotes the velocity of the fluid particle in the direction of the normal $N$. The last integral thus expresses the rate at which the pressure $\Delta p$, exerted from outside an element $\delta\sigma$ of the boundary, is doing work. Assuming the plate to be the fluid–solid interface (White, 1991), Eq. (2) can be expressed as

$$\frac{\mathrm{d}E}{\mathrm{d}t} = -\int_0^H \int_0^l \frac{\partial h}{\partial t}\Delta p \,\mathrm{d}s\,\mathrm{d}y, \tag{3}$$

where $\mathrm{d}s = \sqrt{1+(\partial h/\partial x)^2}\,\mathrm{d}x$ and $\partial h/\partial t$ is the velocity of the plate in the normal direction. Note that the use of $\partial h/\partial t$ for normal velocity implies that the interface is more or less parallel to the $x-y$ plane for sufficiently small oscillations.

On the other hand, the rate of change of kinetic energy in time of the oscillating plate having a mass per unit area equivalent to the added mass, $M$, can be written as (Minami, 1998)

$$\frac{\mathrm{d}E}{\mathrm{d}t} = M\int_0^H \int_0^l \frac{\partial h}{\partial t}\frac{\partial^2 h}{\partial t^2}\,\mathrm{d}s\,\mathrm{d}y. \tag{4}$$

Equating the right-hand sides of Eqs. (3) and (4) one obtains an expression for an added mass per unit area in the form

$$M = -\frac{\int_0^H \int_0^l \Delta p(\partial h/\partial t)\,\mathrm{d}s\,\mathrm{d}y}{\int_0^H \int_0^l (\partial h/\partial t)\,(\partial^2 h/\partial t^2)\,\mathrm{d}s\,\mathrm{d}y}. \tag{5}$$

Eq. (5) displays the essential dependence of the added mass of the oscillating plate upon the fluid loading, $\Delta p$, as will be seen in the analysis below.

## 3. Review of fluid loading formulations

Lord Rayleigh (1879) has shown that for traveling wave solutions, the fluid loading on a unit area of an infinite horizontal surface is

$$\Delta p(\bar{x}, t) = \frac{2\rho}{k}\left(\frac{\partial}{\partial t} + \frac{U}{l}\frac{\partial}{\partial \bar{x}}\right)^2 h(\bar{x}, t), \tag{6}$$

where the fluid has a constant velocity $U$ parallel to the $x-y$ plane (Fig. 2); $t$ is the time; $k = 2\pi/\lambda$ is the wave number; $\lambda$ is the wavelength; $h(x, t)$ and $l$ are the surface displacement and length, respectively, of the plate as defined above. In Eq. (6) the coefficient $2\rho/k$ plays the role of an added mass per unit area, $M$.

For a finite length plate the fluid loading will be assumed to be that for an infinitely long wavy plate of a sinusoidal shape given by (Dugundji et al., 1963)

$$h(\bar{x}, t) = \sum_{n=1}^{\infty} q_n(t)\sin(n\pi\bar{x}), \quad n = 1, 2, 3, \ldots, \tag{7}$$

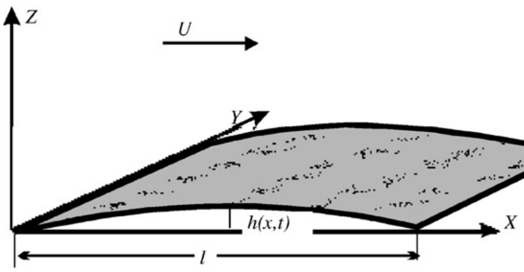


Fig. 2. Model of the plate simply supported at both ends.

where $n$ is the mode number, and $q_n(t)$ is a function of time. This is a reasonable assumption except in the immediate vicinity of the leading and trailing edges, particularly for the high modes (large $n$). Using expression (7), the coefficient of the added mass, $M$, in Eq. (6) reduces to

$$M = \frac{2\rho l}{n\pi}. \tag{8}$$

For a thin narrow plate Wu (1971) has shown that in limits of the slender-wing theory (Jones, 1946) the fluid loading is

$$\Delta p(\bar{x}, t) = \frac{\rho l}{\pi} A \left(\frac{\pi}{2}\right)^2 \left(\frac{\partial}{\partial t} + \frac{U}{l}\frac{\partial}{\partial \bar{x}}\right)^2 h(\bar{x}, t), \tag{9}$$

where $A = H/l$ is the aspect ratio of the plate. The coefficient of added mass here is

$$M = \frac{\rho l}{\pi} A \left(\frac{\pi}{2}\right)^2. \tag{10}$$

This definition is usually referred to as the ‘‘slender-wing approximation’’ and is attributed to Jones (1946). The resulting coefficient of the added mass is identical to that of the circumscribed circular cylinder. However, Sewall et al. (1983) suggested that the added mass determined using the above method provides an overestimate.

Ellen (1973) found an asymptotic form for the pressure for low aspect ratio plates. The fluid loading (averaged across the plate) was found to be

$$\Delta p(\bar{x}, t) = -\frac{2\rho l}{\pi} A \log A \left(\frac{\partial}{\partial t} + \frac{U}{l}\frac{\partial}{\partial \bar{x}}\right)^2 \times (\bar{x}, t) \int_0^1 h_2(\bar{y})\, \mathrm{d}\bar{y}, \tag{11}$$

where the total surface deflection is considered as a product of two independent deflections in the streamwise and the spanwise directions: $h(\bar{x}, \bar{y}, t) = h_1(\bar{x}, t) h_2(\bar{y})$. With the assumption that the spanwise surface deflection may be written as

$$h_2(\bar{y}) = \sin \pi \bar{y}, \quad 0 \leqslant \bar{y} \leqslant 1, \tag{12}$$

Eq. (11) is reduced to

$$\Delta p(\bar{x}, t) = -\frac{\rho l}{\pi} \left(\frac{4}{\pi} A \log A\right) \left(\frac{\partial}{\partial t} + \frac{U}{l}\frac{\partial}{\partial \bar{x}}\right)^2 h_1(\bar{x}, t). \tag{13}$$

The added mass is thus

$$M = \frac{\rho l}{\pi} \left(\frac{4}{\pi} A |\log A|\right). \tag{14}$$

Considering a flexible plate of infinite width and finite length $l$, embedded in an infinite rigid plane, with a uniform incompressible two-dimensional flow, Kornecki et al. (1976) have shown that the flow loading per unit area of the plate may be expressed as

$$\Delta p(\bar{x}, t) = -\frac{2\rho l}{\pi} \left\{ \int_0^1 \left(\frac{\partial}{\partial t} + \frac{U}{l}\frac{\partial}{\partial \xi}\right)^2 \times h(\xi, t) \ln|\bar{x} - \xi| \, \mathrm{d}\xi - R(\bar{x}) \right\}, \tag{15}$$

where

$$R(\bar{x}) = \frac{U}{l}\left(\frac{\partial h}{\partial t}\bigg|_{\bar{x}=1} + \frac{U}{l}\frac{\partial h}{\partial \bar{x}}\bigg|_{\bar{x}=1}\right)\ln(1-\bar{x}) - \frac{U}{l}\left(\frac{\partial h}{\partial t}\bigg|_{\bar{x}=0} + \frac{U}{l}\frac{\partial h}{\partial \bar{x}}\bigg|_{\bar{x}=0}\right)\ln(\bar{x}).$$

Eq. (15) resembles Eqs. (6)–(13). The terms

$$\frac{\partial^2 h}{\partial t^2},\quad 2\frac{U}{l}\frac{\partial^2 h}{\partial \bar{x}\partial t},\quad \left(\frac{U}{l}\right)^2\frac{\partial^2 h}{\partial \bar{x}^2}$$

express the translatory, Coriolis and centrifugal accelerations of the fluid particles, respectively, and the function $\ln|\bar{x}-\xi|$ represents the effect of spatial memory. Considering the explicit terms, one can deduce that the fluid loading has the effects of modifying the added mass, the fluid dynamic damping and the added stiffness on the plate (Huang, 1995). It is evident from Eq. (15) that at points $\bar{x}=0$ and 1 there exist logarithmic singularities. These singularities are versions of the well-known leading and trailing edge singularities of thin airfoil theory (Garrad and Carpenter, 1982b). Kornecki et al. (1976) noted the existence of these singularities but regarded them as unimportant in practice. Subsequently, Garrad and Carpenter (1982a) showed that these singularities are very weak, and that Kornecki et al. (1976) were indeed fully justified in ignoring them.

The generalized fluid loading (Eq. (15)) has been evaluated numerically (Garrad and Carpenter, 1982b) for the plate simply supported at both ends (Fig. 2) by using six sine modes in the Galerkin expansion. The results obtained demonstrate the variation of the aerodynamic inertia with the mode number. By neglecting viscous effects and using a three-dimensional thin wing theory, Lucey and Carpenter (1993) have obtained the generalized fluid loading on the flexible plate as

$$\Delta p(\bar{x},\bar{y},t) = \frac{\rho l}{\pi}\left[F_I\left(\frac{\partial^2 h}{\partial t^2}\right) + UF_D\left(\frac{\partial h}{\partial t}\right) + U^2F_S(h)\right], \tag{16}$$

where

$$F_I\left(\frac{\partial^2 h}{\partial t^2}\right) = A\int_0^1\int_0^1 \frac{1}{R}\frac{\partial^2 h}{\partial t^2}\bigg|_{(\bar{x},\bar{y})=(\xi,\eta)} \mathrm{d}\xi\,\mathrm{d}\eta, \tag{17}$$

$$F_D\left(\frac{\partial h}{\partial t}\right) = \frac{A}{l}\int_0^1\int_0^1 \left[\frac{1}{R}\frac{\partial^2 h}{\partial \xi\partial t}\bigg|_{(\bar{x},\bar{y})=(\xi,\eta)} + \frac{\partial}{\partial \xi}\left(\frac{1}{R}\right)\frac{\partial h}{\partial t}\bigg|_{(\bar{x},\bar{y})=(\xi,\eta)}\right]\mathrm{d}\xi\,\mathrm{d}\eta, \tag{18}$$

$$F_S(h) = \frac{A}{l^2}\int_0^1\int_0^1 \frac{\partial}{\partial \xi}\left(\frac{1}{R}\right)\frac{\partial h}{\partial \xi}\bigg|_{(\bar{x},\bar{y})=(\xi,\eta)} \mathrm{d}\xi\,\mathrm{d}\eta \tag{19}$$

and

$$R = \sqrt{(\bar{x}-\xi)^2 + A^2(\bar{y}-\eta)^2}.$$

Three contributions to the fluid loading (Eq. (16)) are identified as aerodynamic inertia $F_I$, damping $F_D$ and stiffness $F_S$ (Kornecki et al., 1976). These integrals have been evaluated numerically by using three sine modes (Lucey and Carpenter, 1993) for a plate simply supported at all ends and with aspect ratio $A = 1.0$ and 5.0. The results obtained by Lucey and Carpenter (1993) illustrate how the aerodynamic inertia varies with the mode number and the aspect ratio.

## 4. Results and discussion of the numerical calculation

To obtain values for the added mass, it is necessary to define the deflection form of the cantilever plate (Eq. (1)). If we assume a half-sine spanwise deflection, the following deflection form can be assumed:

$$h(\bar{x},\bar{y},t) = a(t)\sin\pi\bar{y}\left[\cosh\beta_n\bar{x} - \cos\beta_n\bar{x} - (\sinh\beta_n\bar{x} - \sin\beta_n\bar{x})\frac{\cosh\beta_n + \cos\beta_n}{\sinh\beta_n + \sin\beta_n}\right], \tag{20}$$

where the eigenvalues, $\beta_n$, are (Paidoussis, 1998):

$$\beta_n = 1.8751, 4.69409, 7.85476, \text{etc.}, \quad n = 1, 2, 3\ldots .$$

For small oscillations of the plate, the length of the element d$s$ can be written as d$s \approx$ d$x$, and Eq. (5) takes the following form:

$$M = -\frac{\int_0^H \int_0^l \Delta p(\partial h/\partial t)\,\mathrm{d}x\,\mathrm{d}y}{\int_0^H \int_0^l (\partial h/\partial t)(\partial^2 h/\partial t^2)\,\mathrm{d}x\,\mathrm{d}y}. \tag{21}$$

The values of the added mass per unit area are calculated from Eqs. (20) and (21) for cantilever plates with various aspect ratios. The expression for the fluid loading $\Delta p$, in Eq. (21), is chosen according to a three-dimensional aerodynamic theory (Eq. (16)) for a stationary fluid (i.e., $U = 0$, see Appendix A, Eq. (A.1)). Direct numerical integration is used in the solution of Eq. (21) for the first 10 chordwise natural modes ($n = 1, 2, \ldots, 10$) in expression (20).

It should be noted that all the expressions for the fluid loading presented in Appendix A have the same multiplier $\rho l/\pi$, and so a nondimensional value for the added mass, $\mu = M/(\rho l/\pi)$, will be used.

Values for the added mass (stationary fluid, $U = 0$) were also calculated according to the two-dimensional aerodynamic theory, the slender-wing approximation for aspect ratio $A = 0.1$ and the travelling wave solution (approximation of Dugundji et al. (1963)) for an infinitely wide plate (see Eqs. (A.2)–(A.4) in Appendix A, respectively).

The dependence of the added mass, $\mu$, on the number of the natural mode, $n$, is shown in Fig. 3. The largest variation of the added mass with aspect ratio occurs in the lower modes. The dependence of the added mass on the order of the chordwise natural mode diminishes as the number $n$ increases. As a result, the value of the added mass of a cantilever plate vibrating in a high natural mode, calculated by three-dimensional equation, approaches asymptotically the value of the added mass calculated by the travelling wave solution for the same natural mode, $n$ (Eq. (8)). It is clear that for low aspect ratio and for the low modes an appreciable discrepancy exists between the values calculated by the three-dimensional aerodynamic theory and those calculated by the slender-wing theory (Eq. (10)).

An evaluation of the present method for calculating the added mass has been performed for a plate simply supported on all sides. The following expression for the displacement of the vibrating plate is used in place of Eq. (20):

$$h(\bar{x}, \bar{y}, t) = a(t)\sin n\pi\bar{x}\,\sin\pi\bar{y},$$
$$0 \leqslant \bar{x} \leqslant 1, \quad 0 \leqslant \bar{y} \leqslant 1, \quad n = 1, 2, 3, \ldots . \tag{22}$$

The expressions for the fluid loading, $\Delta p$, for the simply supported plate is the same as those used for the cantilever plate. The calculated values of the nondimensional added mass are presented in Fig. 4.

For the verification of the present results, calculations were carried out for the added mass values, for a three-dimensional problem, allowing only three spanwise/chordwise natural modes following Lucey and Carpenter (1993) for aspect ratio $A = 1.0$, and for a two-dimensional problem allowing six chordwise natural modes following Garrad and Carpenter (1982b). The comparisons are presented in Fig. 4. Also presented is one value for a two-dimensional problem with only one half-sine chordwise natural mode (Minami, 1998).

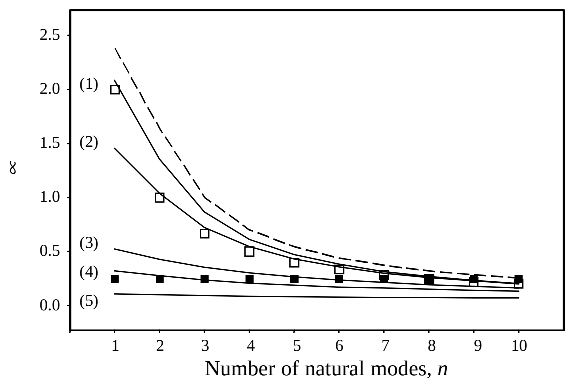


Fig. 3. The dependence of the added mass, $\mu$, on both the aspect ratio, $A$, and the number of the natural mode, $n$, of the cantilever plate in still fluid. 2-D theory (dotted line); 3-D theory (continuous lines): (1) $A = 1.0$; (2) $A = 0.5$; (3) $A = 0.1$; (4) $A = 0.05$; (5) $A = 0.01$. For comparison: (■) slender-wing approximation, $A = 0.1$ (Jones, 1946); (□) travelling wave solution (Rayleigh, 1879).

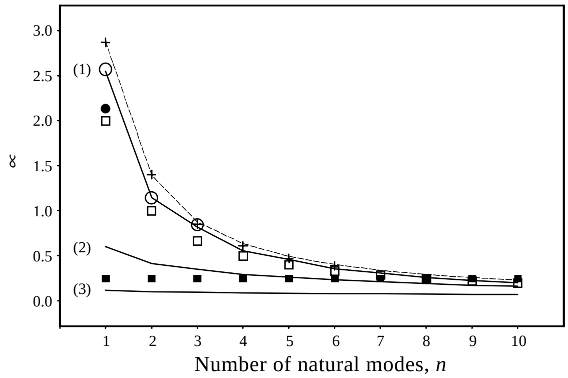


Fig. 4. Dependence of the added mass, $\mu$, on both the aspect ratio, $A$, and the number of the natural mode, $n$, of the simply supported plate in still fluid. 2-D theory (dotted line); 3-D theory (continuous lines): (1) $A = 1.0$; (2) $A = 0.1$; (3) $A = 0.01$. For comparison: (■), slender-wing approximation, $A = 0.1$ (Jones, 1946); (□) travelling wave solution (Rayleigh, 1879); (+) 2-D theory (Garrad and Carpenter, 1982b); (○) 3-D theory, $A = 1.0$, (Lucey and Carpenter, 1993); (●) thin airfoil theory (Minami, 1998).

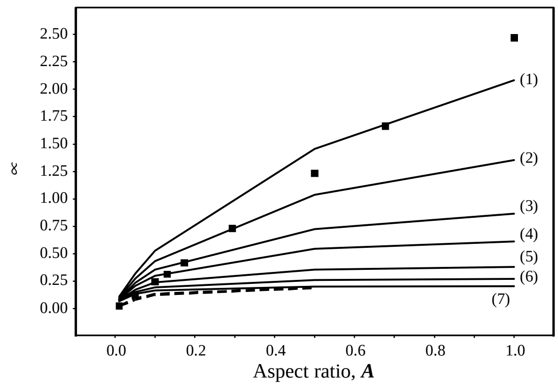


Fig. 5. The variation of the added mass, $\mu$, with the aspect ratio, $A$, of the cantilevered plate in still fluid for various single natural modes, $n$, according to the present 3-D analysis: (1) $n = 1$; (2) $n = 2$; (3) $n = 3$; (4) $n = 4$; (5) $n = 6$; (6) $n = 8$; (7) $n = 10$. For comparison: (■) slender-wing approximation (Jones, 1946); (- - -) low aspect ratio approximation (Ellen, 1973).

The comparison between the present results and those of Lucey and Carpenter (1993) shows a good agreement. It is interesting to see from the analysis of Figs. 3 and 4 that the values of the added mass for the simply supported plate are higher than those for the cantilever plate for similar values of modal index $n$.

Fig. 5 presents the dependencies of the added mass on the aspect ratio and the number of the natural mode of the cantilever plate. The results show that, assuming a single natural chordwise mode oscillation for a three-dimensional problem, there is for the lowest four modes a strong dependence of the added mass on the aspect ratio.

The results obtained by the present solution of the three-dimensional problem can also be compared with those of the slender-wing theory (Jones, 1946, Eq. (A.3)) and Ellen's approximation (Ellen, 1973, Eq. (A.5)) in Fig. 5. The comparison shows that the applicability of the slender-wing approximation depends upon both the mode number and the aspect ratio. In particular, the added mass defined by the slender-wing approximation is overestimated for the first natural mode oscillation ($n = 1$) and for aspect ratios larger than $\approx 0.7$; while for aspect ratios less than $\approx 0.7$ the results are overestimated for higher mode numbers and underestimated for the first natural mode oscillation. Ellen's approximation compares well with our results for high natural mode numbers. Furthermore, we see that the slender-wing approximation does not become more accurate at very low aspect ratios ($A \approx 0.1$); and it underestimates the values of the added mass, especially for plates vibrating in the fundamental mode.

## 5. Conclusions

A three-dimensional analysis for calculating the added mass of a cantilever plate vibrating in a single mode is presented. The approach assumes a spanwise half-sine fundamental mode and a single natural mode in the chordwise direction. The thin airfoil theory for an incompressible fluid is applied. The main findings are: (a) the new approach is in good agreement with earlier results for a simply supported vibrating plate, which was chosen as the verification test case, (b) the nondimensional added mass is a function of the plate's aspect ratio and the order of the natural mode of vibration, (c) increase of the order of the chordwise natural mode of vibration decreases the value of the added mass, (d) decrease of the aspect ratio leads to a decrease of the added mass and attenuates the dependence of the added mass on the order of the chordwise natural mode. This dependence diminishes as the aspect ratio approaches 0.01. When the aspect ratio equals unity, the results obtained by a three-dimensional theory are close to those calculated by a two-dimensional approach ($A = \infty$), (e) the commonly used slender-wing approximation for calculating the added mass tends to overestimate the results for aspect ratios higher than $\approx$0.7, and underestimates these values for lower aspect ratio plates vibrating in the fundamental mode.

## Acknowledgements

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## Appendix A

The expressions for fluid loading (stationary fluid, $U = 0$) may be summarized as follows.

Lucey and Carpenter (1993):

$$\Delta p = \frac{\rho l}{\pi} A \int_0^1 \int_0^1 \left.\frac{\partial^2 h}{\partial t^2}\right|_{(\bar{x},\bar{y})=(\xi,\eta)} \times \frac{\mathrm{d}\xi\,\mathrm{d}\eta}{\sqrt{(\bar{x}-\xi)^2 + A^2(\bar{y}-\eta)^2}}; \tag{A.1}$$

Kornecki et al. (1976), and Garrad and Carpenter (1982b):

$$\Delta p = -\frac{2\rho l}{\pi} \int_0^1 \frac{\partial^2 h}{\partial t^2} \ln|\bar{x}-\xi|\,\mathrm{d}\xi; \tag{A.2}$$

Jones (1946) and Wu (1971):

$$\Delta p = \frac{\rho l}{\pi} A \left(\frac{\pi}{2}\right)^2 \frac{\partial^2 h}{\partial t^2}; \tag{A.3}$$

Rayleigh (1879), Miles (1956), and Dugundji et al. (1963):

$$\Delta p = \frac{\rho l}{\pi} \frac{2}{n} \frac{\partial^2 h}{\partial t^2} \quad n = 1, 2, 3, \ldots; \tag{A.4}$$

Ellen (1973):

$$\Delta p = -\frac{\rho l}{\pi} \left(\frac{4A \log A}{\pi}\right) \frac{\partial^2 h}{\partial t^2}. \tag{A.5}$$

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